

Lec 11: Advanced DP technique

Want :- space complexity
Time "

$$\frac{O(\min(m,n))}{O(m)} = O(m,n) = am + bn$$

actual seq.

$$O(mn)$$

X →	A	L	G	O	R	I	T	H	M	m
	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	3	4	5	6	6
U	5	4	3	3	3	3	4	5	6	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

$$\text{Edit}(i,j) = \text{Edit dist. } (X[1 \dots i], Y[1 \dots j])$$

$$1) \text{ Edit}(i,j) = \text{Edit}(i-1, j-1)$$

$$\text{Edit}(2,2) = \text{Edit}(1,1)$$

$$AL_x \rightsquigarrow AL_y \equiv [A_x \rightsquigarrow A_y]$$

$$2) \text{ Edit}(i,j) = \text{Edit}(i-1, j-1) + 1$$

$$\text{Edit}(4,3) = \text{Edit}(3,2) + 1$$

$$ALGO \rightsquigarrow ALT \equiv (ALG \rightsquigarrow AL) + [\text{change } O \rightarrow T]$$

$$3) \text{ Edit}(i,j) = \text{Edit}(i-1, j) + 1$$

$$\text{Edit}(6,3) = \text{Edit}(5,3) + 1$$

$$ALGORI \rightsquigarrow ALT \equiv \text{delete } I$$

$$ALG \rightsquigarrow ATRUI \equiv \text{inc. } I + ALGOR \rightsquigarrow ALT$$

$$4) \text{ Edit}(i,j) = \text{Edit}(i, j-1) + 1$$

$$\text{Edit}(3,5) = \text{Edit}(3,4) + 1$$

→ deletion
↓ insertion
→ replacement
→ no change

$$\text{Edit}(i,j) =$$

base case

$$\min (a) \text{ Edit}(i-1, j) + 1$$

delete $X[i]$

$$(b) \text{ Edit}(i, j-1) + 1$$

insert $Y[j]$

$$(c) \text{ Edit}(i-1, j-1) + 1$$

if $X[i] \neq Y[j]$

$$(d) \text{ Edit}(i-1, j-1)$$

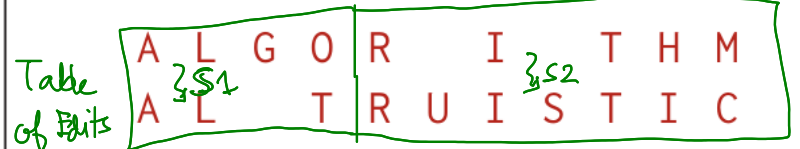
change
if $X[i] = Y[j]$
no change

$\text{Edit}(3,2) =$ number of edits for $ALG \rightarrow AL$

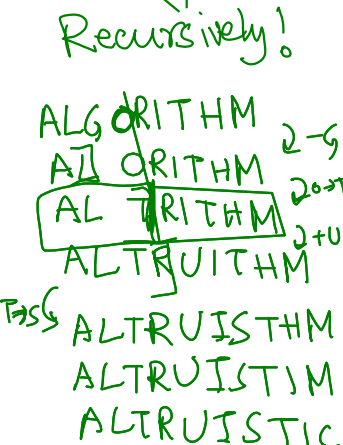
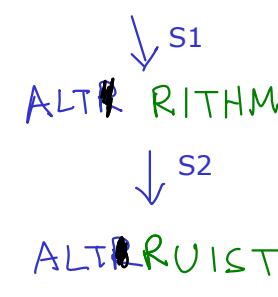
$$= \begin{cases} 1 (\text{insertion of } L) & + \text{ number of edits for } ALG \rightarrow A \\ 1 (\text{deletion of } G) & + \text{ " " " " } AL \rightarrow AL \end{cases}$$

delete G: 1 + Edit(2,2)
 insert L: 1 + Edit(3,1)
 change G → L: 1 + Edit(3,3)

	A	L	G	O	R	I	T	H	M	
0	→1	→2	→3	→4	→5	→6	→7	→8	→9	
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
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- Find h s.t. ALGO is changed to ALT[1 ... h] in the opt. edit sequence from ALGORITHM → ALTRISTIC
- Find one optimal sequence for ALGO → ALT[1 ... h]: S_1
- Find one optimal sequence (reuse ^{memo} space) for RITHM → ALT[h+1 ... 10]: S_2
- return ALGORITHM

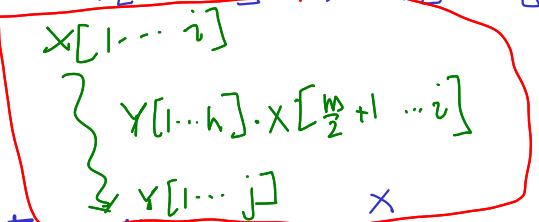


$h = \min h(m, n)$

① How to find $h = \text{Half}(X, Y)$

$X_m \rightarrow Y_n$
 $-G, O \rightarrow T, *U \dots$

Define $h(i, j)$ = length of the first part of Y to which $X[1 \dots \frac{i}{2}]$ is changed to, in some optimal edit sequence of $X[1 \dots i] \rightarrow Y[1 \dots j]$, when $i \geq \frac{m}{2}$; $h(i, j) = \infty$ otherwise



Examples

ALGO → ALTR
 A → A
 AL → AL
 ALG → ALT
 ALGO → ALTR end → A

ALG is edited to A
 (A)G ← A

$h(3, 1) = ? 1$
 In the optimal edit sequence from ALGO[1...3] → ALTR[1]

ALGO[1...2] is edited to ALTR[1...h(3,1)]

$$h^{m,n}(i,j) =$$

{ undefined ∞ if $i > m/2$
 base case j if $i = m/2$
insertion $h^{m,n}(i, j-1)$ if
deletion $h^{m,n}(i-1, j)$ if
change $h^{m,n}(i-1, j-1)$ if
no change $h^{m,n}(i-1, j-1)$ if

$h^{10,9}(2,8)$
 $= AL \rightsquigarrow ALTRUIST$
 where does ALGOR map to?
 $h^{10,9}(5,4) = ALGOR \rightsquigarrow ALTR$
 where does ALGOR map to?
 $ALTR$
 $h^{10,9}(i,j)$
 $h^{10,9}(2,8)$
 AL
 ↓
 AL...T
 ALGOR

$$Edit^{m,n}(i,j) = Edit(i, j-1) + 1$$

$$Edit^{m,n}(i,j) = Edit(i-1, j) + 1$$

$$Edit^{m,n}(i,j) = Edit(i-1, j-1) + 1$$

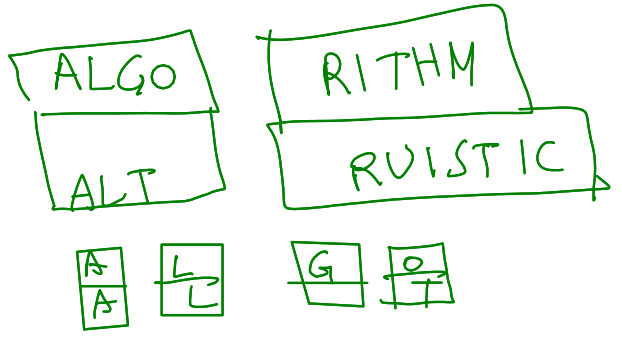
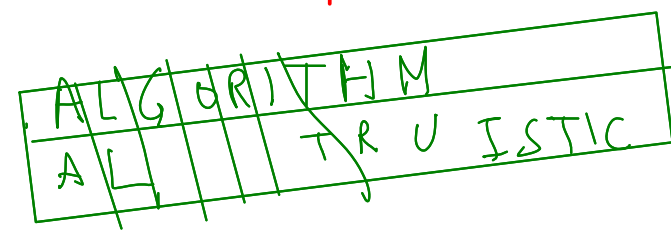
$$Edit^{m,n}(i,j) = Edit(i-1, j-1)$$

compute $Edit(i,j)$ & $h(i,j)$ at the same time

	A	L	G	O	R	I	T	H	M
0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7
L	2	1	0	1	2	3	4	5	6
T	3	2	1	1	2	3	4	4	5
R	4	3	2	2	2	2	3	4	5
U	5	4	3	3	3	3	3	4	5
I	6	5	4	4	4	4	3	4	5
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$Edit_{X,Y}(6,3) = Edit_{X,Y}(5,2) + 1 \rightarrow$ change
 \Rightarrow ALGOR I
 $ALGOR \rightsquigarrow AL \leftarrow$ matching ALT
 $Edit_{X,Y}(6,3) = Edit_{X,Y}(5,3) + 1$
 \Rightarrow
 $Edit_{X,Y}(9,10) = ?$

Opt. space complexity. to
 compute $h^{m,n}(m,n) = h$ (needed for 1st step)
 $= O(\min(m,n))$



② How to find optimal ^{edit} seq from $X' \rightarrow Y'$

a. Find $h^{m,n}(m,n) \rightarrow h' = \text{Half}(X', Y')$

b. Recursively compute an optimal edit sequence for $X'[1 \dots \frac{m}{2}]$
 $E_1 \downarrow$
 $Y'[1 \dots h']$

c. Recursively compute an optimal edit sequence for $X'[\frac{m}{2}+1 \dots m]$
 $E_2 \downarrow$
 $Y'[h'+1 \dots n]$

d. Base case: If $m=1$? \rightarrow Regular Dyn. Prog - $O(n)$ space
 If $n=1$? \rightarrow " " " $O(m)$ space

e. Return (E_1, E_2)

③ Complexity analysis

Space complexity $\rightarrow S(m,n) = 2m + \max\{S(\frac{m}{2}, h'), S(\frac{m}{2}, n-h')\}$
 Prove by ind. $S(m,n) \leq cm + dn$ for some $c \geq d$

compute h, needs 2 rows only

Time complexity $\rightarrow T(m,n) = O(mn) + \max_h T(\frac{m}{2}, h) + T(\frac{m}{2}, n-h)$
 Prove by ind. $T(m,n) \leq t mn$

compute h