Lec 11: Advanced DP technique

Want :- space complexity $O(\min(m,n)) \Rightarrow O(m,n) = am+bn$ ~~O(m)~~ actual eq.

Time " $O(mn)$

$Edit(i,j) = $ Edit dist. $(X[1\cdots i], Y[1\cdots j])$

Y→ \ X→		A	L	G	O	R	I	T	H	M
	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

m

n

→ deletion

↓ insertion

↘ replacement

↘ no change

1) Edit(i,j) = Edit(i-1,j-1)

Edit(2,2) = Edit(1,1)

$AL_x \rightsquigarrow AL_Y \equiv [A_x \rightsquigarrow A_Y]$

2) Edit(i,j) = Edit(i-1,j-1) + 1

Edit(4,3) = Edit(3,2) + 1

2   1

ALGO ⇝ ALT ≡ (ALG ⇝ AL) + [change O → T]

3) Edit(i,j) = Edit(i-1,j) + 1

Edit(6,3) = Edit(5,3) + 1

4   3

ALGORI ⇝ ALT ≡ delete I + ALGOR ⇝ ALT

ALG ⇝ ATRUI ≡ ins. I + ALG ⇝ ALTRU

4) Edit(i,j) = Edit(i,j-1) + 1

Edit(3,5) = Edit(3,4) + 1

3   2

$X_1 \cdots X_{i-1}\ X_i$

$Y_1 \cdots Y_{j-1}\ Y_j$

$$Edit(i,j) = \begin{cases} \text{base case} \\ \min \begin{cases} a)\ Edit(i-1,j)+1 & \text{delete } X[i] \\ b)\ Edit(i,j-1)+1 & \text{insert } Y[j] \\ c)\ Edit(i-1,j-1)+1 & \text{if } X[i] \neq Y[j] \text{ change} \\ d)\ Edit(i-1,j-1) & \text{if } X[i] = Y[j] \text{ no change} \end{cases} \end{cases}$$

Edit(3,2) = number of edits for ALG → AL

= { 1 (insertion of L) + number of edits for ALG → A

{ 1 (deletion of G) + " " " " AL → AL

		A	L	G	O	R	I	T	H	M
	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

m

n

$h = h^{m,n}(m,n)$

Table of Edits

Edit(3, 2) = delete G: 1 + Edit(2,2)
insert L: 1 + Edit(3,1)
change G → L: 1 + Edit(2,1)

A L G O	R I T H M
A L T	R U I S T I C

S1, S2

ALG → AL
ALGO → ALT
ALGOR → ALTR

h=3 in this ex.

1. Find h s.t. ALGO is changed to ALT[1 ... h] in the opt. edit sequence from ALGORITHM -> ALTRISTIC

2. Find one optimal sequence for ALGO -> ALT[1 ... h]: S1

3. Find one optimal sequence (reuse space) for RITHM -> ALT[h+1 ... 10] : S2
memo

4. return ALGORITHM

↓ S1

ALT~~R~~ RITHM

↓ S2

ALT~~R~~RUISTIC

Recursively!

ALGORITHM
AL ORITHM   2→G
AL TRITHM   2O→T
ALTRUITHM   2+U
ALTRUISTHM
ALTRUISTIM
ALTRUISTIC

−G, O→T, +U ...

① How to find h = Half(X,Y)

$X_{(m)} \rightarrow Y_{(n)}$

Define $h^{m,n}(i, j)$ = length of the first part of Y to which X[1 ... m/2] (not i/2) is changed to, in some optimal edit sequence of X[1 ... i] → Y[1 ... j], when $i \geq \frac{m}{2}$; $h^{m,n}(i,j) = \infty$ otherwise
|X| |Y|

X[1 ... i]

Y[1...h].X[m/2 + 1 ... i]

Y[1 ... j]

ALG

Examples

X → Y
ALGO → ALTR

A → A
AL → AL
ALG → ALT
ALGO → ALTR end →

start → ALG
ALG
A G
A

AL is edited to A

$h^4(3, 1) = ?\ 1$

In the optimal edit sequence from ALGO[1...3] → ALTR[1]
ALG → A

ALGO[1...2] is edited to ALTR[1... h(3,1)]
AL A

$$h^{m,n}(i,j) = \begin{cases} \text{undefined} & \infty & \text{if } \cancel{i > m/2} \quad i < m/2 \\ \text{base case} & j & \text{if } i = m/2 \\ \text{insertion} & h^{m,n}(i,j-1) & \text{if } Edit^{m,n}(i,j) = Edit(i,j-1)+1 \\ \text{deletion} & h^{m,n}(i-1,j) & \text{if } Edit^{m,n}(i,j) = Edit(i-1,j)+1 \\ \text{change} & \boxed{h^{m,n}(i-1,j-1)} & \text{if } Edit^{m,n}(i,j) = Edit(i-1,j-1)+1 \\ \text{no change} & h^{m,n}(i-1,j-1) & \text{if } Edit^{m,n}(i,j) = Edit(i-1,j-1) \end{cases}$$

$h^{10,9}(2,8)$ = AL ⇝ ALTRUIST

where does ALGOR map to?

$h^{10,9}(5,4)$ = ALGOR ⇝ ALTR

where does ALGOR ⇝ ? ALTR

$h^{10,9}(i,j)$

$h^{10,9}(2,8)$

AL

AL... T

ALGOR?

compute Edit(i,j) & h(i,j) at the same time

Y↓ X→		A	L	G	O	R	I	T	H	M
	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
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T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

m, n

Edit_X,Y(6,3) = Edit_X,Y(5,2) + 1 → change

=>

ALGO RI ⇝ ALT

ALGO R ⇝ AL ⟷ matching ALT

Edit_X,Y(6,3) = Edit_X,Y(5,3)+1

=>

Edit_X,Y(9,10) = ?

Opt. space complexity: to compute $h^{m,n}(m,n) = h$ (needed for last step) $= O(\min(m,n))$

ALGORITHM

ALTRUISTIC

ALGO | RITHM

ALT | RUISTIC

A/A L/L G/ O/T

② How to find optimal edit seq from $X' \to Y'$ (with X' of length m, Y' of length n)

a. Find $h^{m,n}(m,n) \to h' = \text{Half}(X', Y')$

b. Recursively compute an optimal edit sequence for $X'[1 \ldots \frac{m}{2}] \overset{E_1}{\leadsto} Y'[1 \ldots h']$

c. Recursively compute an optimal edit sequence for $X'[\frac{m}{2}+1 \ldots m] \overset{E_2}{\leadsto} Y'[h'+1 \ldots n]$

d. Base case: If $m=1$? $\to$ Construct optimal edit sequence using Regular Dyn. Prog. - $O(n)$ space
If $n=1$? $\to$ " " " $O(m)$ space

e. Return (E_1, E_2)

③ Complexity analysis

Space complexity $\to S(m,n) = 2m + \max_{h'} \{ S(\frac{m}{2}, h'), S(\frac{m}{2}, n-h') \}$

Compute h, needs 2 rows only

Prove by ind. $S(m,n) \le cm + dn$ for some c & d

Time complexity $\to T(m,n) = O(mn) + \max_{h} T(\frac{m}{2}, h') + T(\frac{m}{2}, n-h')$

compute h

Prove by ind. $T(m,n) \le tmn$.